DEPARTMENT OF PHYSICS
BROWN UNIVERSITY
Written Qualifying Examination for the Ph.D. Degree
January 25, 2013

READ THESE INSTRUCTIONS CAREFULLY

1. The time allowed to complete the exam is 12:00–5:00 PM.
2. All work is to be done without the use of books or papers and without help from anyone. The use of calculators or other electronic devices is also not permitted.
3. Use a separate answer book for each question, or two books if necessary.
4. **DO NOT write your name in your booklets.** Each student has been assigned a letter code which is on the outside front cover of each exam booklet. This letter code provides anonymity to the student for faculty grading. Please make sure that this letter is listed on ALL exam booklets that you use.
5. Write the problem number in the center of the outside front cover. **Write nothing else** on the inside or outside of the front and back covers. Note that there are separate graders for each question.
6. Answer one (and **only** one) problem from each of the five pairs of questions. The pairs are labeled as follows:

Classical Mechanics	CM1, CM2
Electricity and Magnetism	EM1, EM2
Statistical Mechanics	SM1, SM2
Quantum Mechanics	QM1, QM2
Quantum Mechanics	QM3, QM4

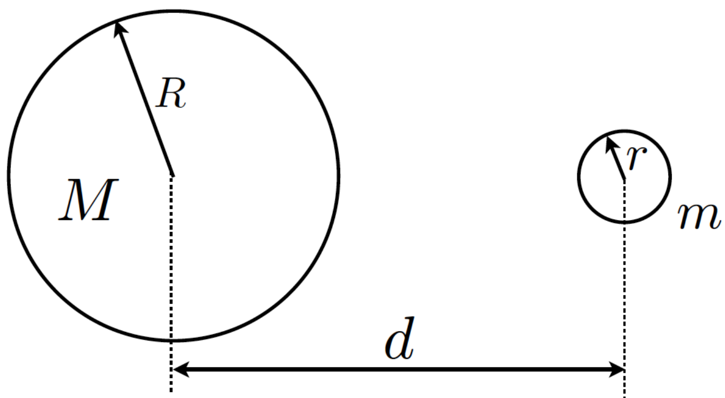
   Note that there are two pairs of Quantum Mechanics problems. You have to do **one** problem from **each** pair.
7. All problems have equal weight.

1. CM-1

There is evidence that our Milky Way galaxy has grown by assimilating nearby dwarf galaxies.

(a) (4 points) Consider a dwarf galaxy of mass m and radius r that is in circular orbit about the Milky Way (mass M). What is the critical distance d between the galaxies centers, smaller than which stars in the dwarf galaxy will be tidally stripped by the Milky Way?

(b) (6 points) Now consider a dwarf galaxy of mass m and radius r that approaches the Milky Way (mass M and radius R) with a velocity v at the distance d of closest approach between their centers. Estimate the maximum value of d such that stars in the dwarf galaxy will be stripped by the Milky Way. Assume that $r \ll d$, and that the tidal forces act for a time $t = d/v$.



2. CM-2

Consider the classical equation of motion for a spin $\vec{S}(t)^2 = s^2$ in a constant external magnetic field

$$\dot{\vec{S}}(t) = \vec{S} \times \vec{B}$$

(a) Use spherical coordinates for $\vec{S}$ to work out the equations for $\theta(t), \phi(t)$ giving the direction of the spin.

(b) Find a Lagrangian for the equations of motion established in (a).

3. EM-1

(a) (4 points) Starting from Maxwell's Equations, derive how the magnitude of the propagation wave vector depends on μ, the permeability, ϵ, the permittivity, c, the speed of light, σ, the conductivity and ω, the angular frequency of the radiation. Presume that μ and ϵ are constants (i.e. linear response).

(b) (4 points) Drude introduced a model for the conductivity of a metal or a plasma that started with an equation of motion for a single charge q of mass m

$$m\ddot{\mathbf{r}} + \Gamma m\dot{\mathbf{r}} = e\mathbf{E}$$

where $\mathbf{r}$ is the position vector of the charge and Γ gives the damping. Show that the conductivity of a metal with N electrons per unit volume is given by:

$$\sigma = \frac{Ne^2/m}{\Gamma - i\omega}$$

(c) (2 points) What is the physical meaning of the imaginary part of σ?

4. EM-2

A thin circular ring of radius r, made of a material with conductivity σ and mass density ρ is completely immersed in a uniform magnetic field $\vec{B}$. The ring rotates around an axis along its diameter that is perpendicular to the magnetic field. Find an expression for the angular frequency ω with which the ring rotates in terms of the quantities given as a function of time t, assuming that it rotates with an angular frequency ω_0 at time $t = 0$ and assuming that the magnitude of the change in the angular frequency during any one rotation period is small compared to the magnitude of the angular frequency.

5. SM-1

Consider an ideal gas in a box of volume V in contact with a heat bath at temperature T. Prove that for both classical ideal gas and quantum ideal gas, the following relationship between pressure (P) and volume (V) holds:

$$P = 2\langle E \rangle / 3V$$

where $\langle E \rangle$ is the internal energy of the ideal gas at temperature T.

(a) (6 points) Derive the proof using quantum mechanics.

(b) (4 points) Derive the proof using classical physics.

6. SM-2

The speed distribution of ideal gas molecules in a container follows the Maxwell distribution $f(v) \propto v^2 e^{-mv^2/2k_BT}$, where v is the speed, m the mass of each molecule, k_B the Boltzmann constant, and T the absolute temperature. The number of molecules with speed v that hit the wall in a given time is proportional to the speed and to $f(v)$.

(a) (4 points) If there is a tiny hole in the wall (too small to have much effect on the distribution inside), write down the speed distribution of those that escape, noted as $F(v)$. For the answer here, do not be concerned with the normalization constant yet.

(b) (6 points) Aided by the speed distribution function, calculate the average energy of those molecules that escape the hole.

7. QM-1

(a) (3 points) Find the transition probability from state s to state n. Calculate $P(s \to n)$ for the perturbation

$$V(t) = A\frac{1}{\pi\tau}e^{-t^2/\tau^2},$$

where A is a time-independent operator, working to first order in V(t).

(b) (3 points) In the limit where $\tau \ll 1/\omega_{ns}$ (where $\hbar\omega_{ns}$ is the energy difference between the two states), what does the transition probability calculated in part (a) reduce to? Show that in this limit the perturbation may be treated as $V(t) = A\delta(t)$.

(c) (4 points) Also show that the wavefunction $\phi(t)$ has a discontinuity at $t = 0$ in this limit, and that the total probability of a tranisition out of state s is gven by

$$P = \frac{1}{\hbar^2}\left[< s|A^2|s > - < s|A|s >^2\right].$$

Hints: you may find this integral useful: $\int_{\infty}^{\infty} dx e^{-x^2} = \sqrt{\pi}$

8. QM-2

(a) (3 points) Calculate the expectation value $E(\Delta) \equiv \langle\psi_\Delta|\hat{H}|\psi_\Delta\rangle$ of the following Hamiltonian (where $m = \hbar = 1$ for simplicity):

$$\hat{H} = -\frac{1}{2}\frac{d^2}{dx^2} + 8x^6 - 6x^2$$

for the case of a (normalized) Gaussian wavefunction that is specified by a single variational parameter, Δ:

$$\psi_\Delta(x) = (\pi\Delta^2)^{-1/4}e^{-x^2/2\Delta^2}\ . \quad (1)$$

The following integrals may be of use:

$$\frac{1}{\sqrt{\pi\Delta^2}}\int_{-\infty}^{\infty} e^{-x^2/\Delta^2}dx = 1$$

$$\frac{1}{\sqrt{\pi\Delta^2}}\int_{-\infty}^{\infty} x^2e^{-x^2/\Delta^2}dx = \frac{1}{2}\Delta^2$$

$$\frac{1}{\sqrt{\pi\Delta^2}}\int_{-\infty}^{\infty} x^6e^{-x^2/\Delta^2}dx = \frac{15}{8}\Delta^6$$

(b) (3 points) Sketch $E(\Delta)$ as a function of Δ, for $0 < \Delta < 1$. Estimate the minimum energy, and the optimal value of Δ corresponding to this minimum.

(c) (3 points) Now consider instead the (non-normalized) wavefunction $\psi_a(x) = e^{-(x/a)^4}$. Show that it is an eigenstate of $\hat{H}$ provided that a is chosen appropriately, and find the corresponding eigenenergy.

(d) (1 point) Is the eigenstate ψ_a as determined in part (c) a ground state of $\hat{H}$? Why or why not?

9. QM-3

Consider a photoelectric effect experiment using sodium as the target metal, and 300nm light. Observations are made of the photocurrent and the maximum kinetic energy of the ejected electrons. With the initial intensity of light, you observe 1000 electrons ejected per second.

(a) (2 points) Describe what you observe as the intensity of the light decreases. Include qualitative plots of the photocurrent and the maximum kinetic energy. Label any important points on your plots.

(b) (2 points) Describe what you observe as you change the color of the light over a broad range. Include qualitative plots of the photocurrent and the maximum kinetic energy. Label any important points on your plots.

(c) (2 points) If we lived in a universe where it was correct to replace the quantum electromagnetic field by a classical external field coupled to the target metal electrons, and all the other laws of physics were the same, how would your observations in parts (a) and (b) change. Explain your reasoning in detail.

(d) (4 points) What inferences can you make about the nature of light? Give at least 2 inferences for part (a) and 2 for part (b). Explain your reasoning in detail.

10. QM-4

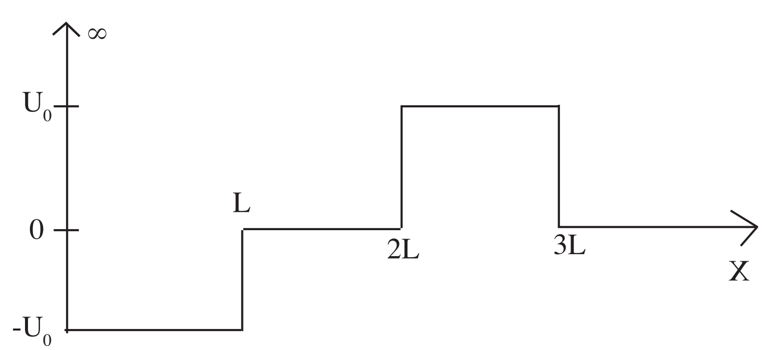
Consider a particle of mass m and energy E that starts at large x in the picture below and moves to the left.

(a) (3 points) Write down the solution for the time-independent Schrodinger equation in piecewise form for all $x > 0$, and $0 < E < U_0$.

(b) (3 points) Write down all of the appropriate boundary conditions and show that you have a sufficient number of equations to determine all of the unknown constants you introduced in part A (no need to solve these equations).

(c) (2 points) Do you expect the energy of this particle to be quantized when $E > 0$? How about if $0 > E > -U_0$, and the particle started in the region $0 < x < L$?

Explain your reasoning.



(d) (2 points) Sketch the wave function for $0 < E < U_0$ for $0 < x < 2L$.