DEPARTMENT OF PHYSICS
BROWN UNIVERSITY
Written Qualifying Examination for the Ph.D. Degree
September, 2018

READ THESE INSTRUCTIONS CAREFULLY

1. The time allowed to complete the exam is 10:00–3:00 PM.
2. All work is to be done without the use of books or papers and without help from anyone. The use of calculators or other electronic devices is also not permitted.
3. Use a separate answer book for each question, or two books if necessary.
4. **DO NOT write your name in your booklets.** Each student has been assigned a letter code which is on the outside front cover of each exam booklet. This letter code provides anonymity to the student for faculty grading. Please make sure that this letter is listed on ALL exam booklets that you use.
5. Write the problem number in the center of the outside front cover. **Write nothing else** on the inside or outside of the front and back covers. Note that there are separate graders for each question.
6. Answer one (and **only** one) problem from each of the five pairs of questions. The pairs are labeled as follows:

Classical Mechanics	CM1, CM2
Electricity and Magnetism	EM1, EM2
Statistical Mechanics	SM1, SM2
Quantum Mechanics	QM1, QM2
Quantum Mechanics	QM3, QM4

   Note that there are two pairs of Quantum Mechanics problems. You have to do **one** problem from **each** pair.

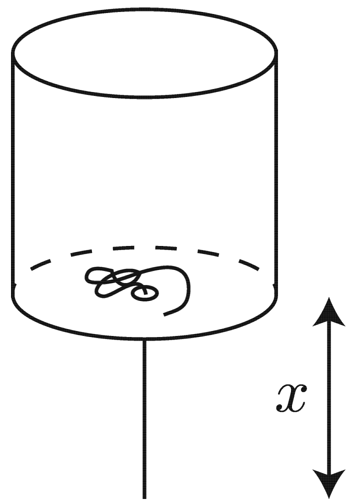
7. All problems have equal weight.

1. CM-1

Consider an infinite chain of particles of mass m. They are connected by springs of the same spring constant β with a uniform spacing of a.

(a) (5 points) Write down equations of motion.

(b) (5 points) Find the frequency ω vs. wavelength q dispersion curves for all possible normal modes.



2. CM-2

One end of a pile of chain falls through a hole in bucket, pulling the remaining links with it one by one. The links are initially at rest and acquire the velocity of the chain suddenly, without friction or resistance from the bucket. The mechanical energy is not conserved.

(a) (5 points)Write the equation of motion.

(b) (5 points) Find the velocity of the chain as a function of x, subject to the initial condition $v = 0$ when $x = 0$. Also, find $x(t)$, where t is time.

Hint: equations of the form

$$\frac{d}{dt}(x\dot{x}) = f(x)$$

can be solved by introducing $y = x^2$ and multiplying by $\dot{y}$.

3. EM-1

A parallel-plate capacitor with plates in the shape of circular disks has the region between its plates filled with a linear dielectric of permittivity ε. Suppose the dielectric has a conductivity σ. Also suppose the capacitor has surface area A, the plates are separated by width d, and the initial charge on it is Q_0. Ignore edge effects.

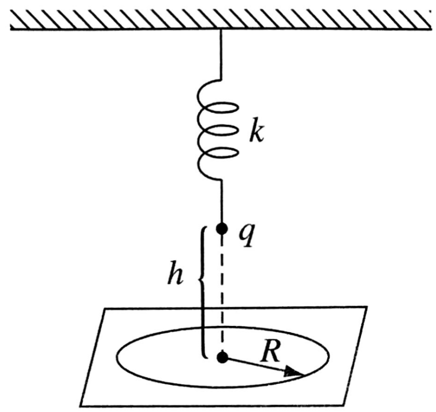
(a) (2 points) Find the charge on the capacitor as a function of time.

(b) (3 points) Find the total displacement current in the dielectric.

(c) (4 points) Find the magnetic field in the dielectric.

(d) (1 point) Provide a qualitative explanation for your answer to part (c).

4. EM-2



A particle of mass m and charge q is attached to a spring with force constant k, hanging from the ceiling (see Figure above). Its equilibrium position is a distance h above the floor. At $t = 0$ it is pulled down a distance d below equilibrium and released.

(a) (5 points) Calculate the intensity of radiation hitting the floor as a function of distance R from the point directly below q in the far-field (i.e., $d \ll \lambda \ll h$). Recall that the time-averaged Poynting vector is

$$\langle \mathbf{S} \rangle = \left(\frac{\mu_0 p_0^2 \omega^4}{32\pi^2 c} \right) \frac{\sin^2 \theta}{r^2} \hat{\mathbf{r}}.$$

(b) (5 points) At what R is the radiation most intense? (Neglect the radiative damping of the oscillator).

5. SM-1

A cubic box with sides of length L contains an ideal gas composed of two different types of partile: $N/2$ particles are of mass m, and the other $N/2$ particles in the box are of mass $m + \delta m$. The box is studied on the surface of Earth, where the gravitational acceleration is g. The temperature T is high enough and the density is low enough that quantum statistics are unimportant.

(a) (5 points) Find the mass density profile inside the box, $\rho(z)$, where z is the height above the bottom of the box. Express your result in terms of N, m, δm, L, g, T, k_B (Boltzmann's constant), and of course z.

(b) (5 points) Could a sensitive measurement of the pressure at the bottom of the box ($z = 0$) distinguish this two-component gas from a different gas of N identical particles, all of which have the mean paricle mass, $m + \frac{1}{2}\delta m$?

Explain your answer by computing the pressure in both cases.

6. SM-2

(a) (4 points)A simple model of a solid consists of a system of N coupled simple harmonic oscillators. Considering only the effect of these oscillators, and without performing any calculations, sketch the specific heat of the classical solid as a function of temperature, call this $C_v(T \to \infty)$. On the same graph, sketch the specific heat of a solid where each of the N harmonic oscillators is treated quantum mechanically, call this $C_v(T)$.

(b) (5 points) Show that the temperature integrated difference between $C_v(T \to \infty)$ and $C_v(T)$ is exactly equal to the zero-point energy of the solid. Hint, write the quantum and classical internal energies in terms of the density of states $g(\omega)$, where ω is the oscillator frequency.

(c) (1 point) Provide a physical interpretation for this result.

7. QM-1

(10 points) For a spin-1 state with an eigenvalue +1 of the spin projection on the axis in the xz plane that makes an angle θ with the z axis, express its wave function as a superposition of S_z eigenstates.

Hint:

$$S^+ = \sqrt{2}\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

8. QM-2

Consider a quantum system which has only three eigenstates. The Hamiltonian in matrix form (i.e. the matrix elements of H with the unperturbed eigenstates) is given by

$$H = V_0 \begin{pmatrix} 1-\varepsilon & 0 & 0 \\ 0 & 1 & \varepsilon \\ 0 & \varepsilon & 2 \end{pmatrix},$$

where V_0 and ε are constants. The unperturbed Hamiltonian corresponds to $\varepsilon = 0$. Assume that $\varepsilon \ll 1$.

(a) (3 points) Find the eigenvectors and eigenvalues of the unperturbed Hamiltonian.

(b) (7 points) Find the first order corrections in ε to the eigenvalues found in part a.

9. QM-3

(10 points) For the hydrogen atom, write down the effective radial Hamiltonian in the sector with angular momentum l. Use the corresponding WKB wavefunction to evaluate the energy levels.

10. QM-4

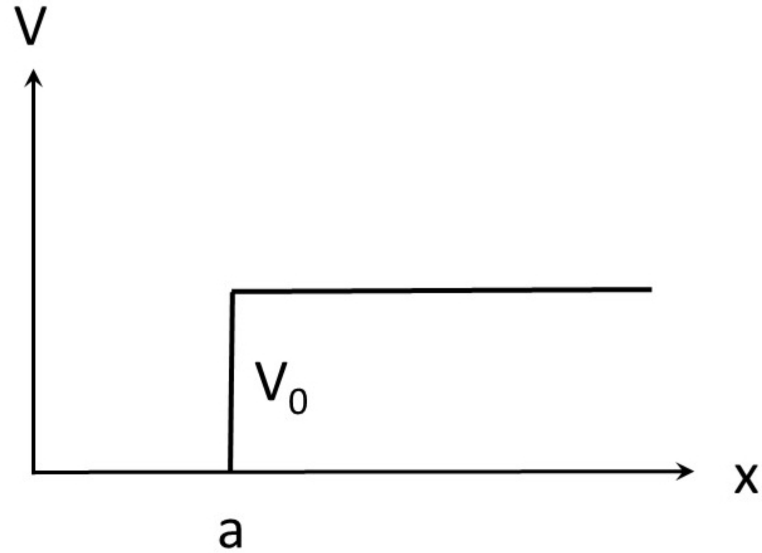
Consider a particle of mass m in a one-dimensional potential $V(x)$ where

$$V(x) = \infty \quad \text{for } x < 0$$

$$V(x) = 0 \quad \text{for } 0 < x < a$$

$$V(x) = V_0 \quad \text{for } x > a$$

and $V_0 > 0$.



(a) (4 points) If $E = \frac{\hbar^2 k^2}{2m}$ is a bound state energy and $E = V_0 - \frac{\hbar^2 K^2}{2m}$, give the equation that determines the possible values of E.

(b) (3 points) What is the condition on V_0 and a for at least one bound state to exist?

(c) (3 points) What are the energy levels when $V_0 = \infty$?