

DEPARTMENT OF PHYSICS
BROWN UNIVERSITY
Written Qualifying Examination for the Ph.D. Degree
August 29, 2014

READ THESE INSTRUCTIONS CAREFULLY

1. The time allowed to complete the exam is 12:00–5:00 PM.
2. All work is to be done without the use of books or papers and without help from anyone. The use of calculators or other electronic devices is also not permitted.
3. Use a separate answer book for each question, or two books if necessary.
4. **DO NOT write your name in your booklets.** Each student has been assigned a letter code which is on the outside front cover of each exam booklet. This letter code provides anonymity to the student for faculty grading. Please make sure that this letter is listed on ALL exam booklets that you use.
5. Write the problem number in the center of the outside front cover. **Write nothing else** on the inside or outside of the front and back covers. Note that there are separate graders for each question.
6. Answer one (and **only** one) problem from each of the five pairs of questions. The pairs are labeled as follows:

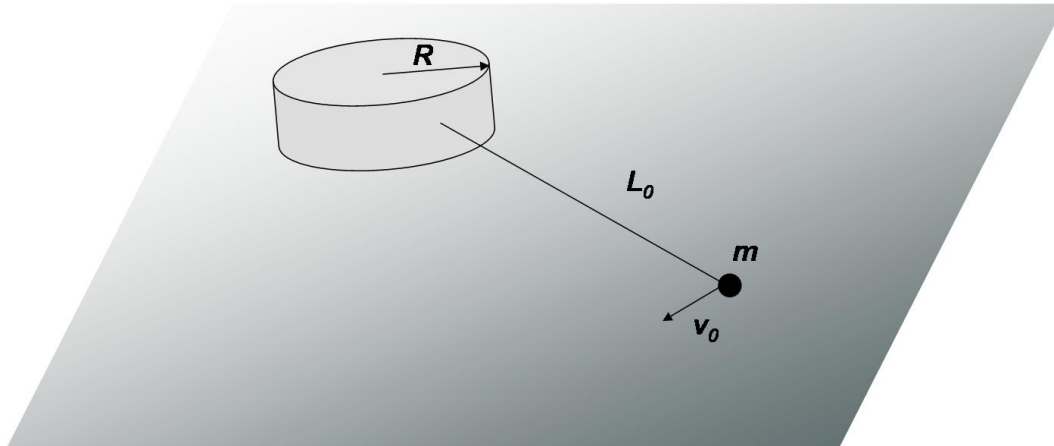
Classical Mechanics	CM1, CM2
Electricity and Magnetism	EM1, EM2
Statistical Mechanics	SM1, SM2
Quantum Mechanics	QM1, QM2
Quantum Mechanics	QM3, QM4

Note that there are two pairs of Quantum Mechanics problems. You have to do **one** problem from **each** pair.

7. All problems have equal weight.

1. CM-1

A small mass m moves on a frictionless horizontal surface. It is tied to a fixed cylinder of radius R with a string of length L_0 . At $t = 0$ the mass is positioned so that the string is tight and radial to the cylinder and the mass moves with a velocity v_0 perpendicular to the direction of the string. Calculate how long it takes for the string to be completely wound around the cylinder and the mass to hit the cylinder.



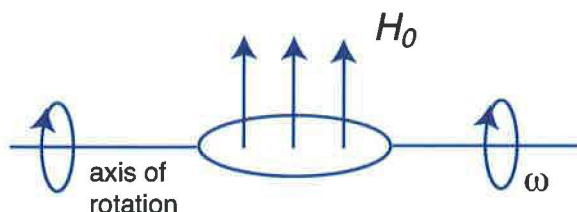
2. CM-2

A point particle moves in two dimensions in the potential $V = kr^2/2$, where $k > 0$. Prove that its trajectory is either an ellipse or a straight line segment.

3. EM-1

A thin copper ring rotates about an axis perpendicular to a uniform magnetic field H_0 . Its initial frequency of rotation is ω . Calculate the time it takes the frequency to decrease to $1/e$ of its original value under the assumption that the energy goes into Joule heating.

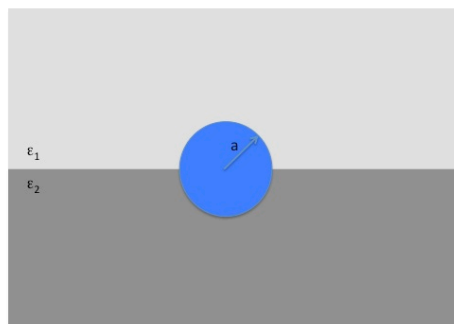
Assume following values: copper conductivity is $\sigma = 5 \times 10^{17}$ cgs and its density $\rho = 8.9 \text{ gm/cm}^3$, and $H_0 = 200$ G.



4. EM-2

Consider a charged metal sphere of radius a . The total charge on the metal sphere is q . Half the sphere is in medium with dielectric constant ϵ_1 , and half in ϵ_2 , as shown below.

- (1) Find the surface charge densities of each half.
- (2) Find the electric field everywhere.
- (3) Find the surface potential of the metal sphere, assuming that the zero of the electric potential is at infinity.
- (4) Find the bound charge density of the dielectric just outside the surface of the sphere.



5. SM-1

Consider a gas of N spin-zero bosons in a d -dimensional box of volume V with a dispersion relation:

$$\epsilon_p = \alpha |\mathbf{p}|^s$$

where the constant α and the index s are both positive.

(a) Find expressions for the mean number of particles per unit volume in the ground state and the mean total number of particles in the excited states in terms of the temperature T and the fugacity $z = \exp(\beta\mu)$. Hint: the volume of a sphere of radius R in d dimensions is $2\pi^{d/2}R^d/(d\Gamma(d/2))$.

(b) Find the condition on s and d for which Bose-Einstein condensation takes place at a finite temperature.

6. SM-2

In this problem, we use a simple lattice model to describe the oxygen molecules in blood solution which accounts for the configurational entropy. In this model, a total of N oxygen molecules can be distributed among Ω sites with $\Omega \gg N$ corresponding to a dilute solution. This is the zero energy reference state for the model. The oxygen molecules can bind to a single dimoglobin molecule (a simplified version of hemoglobin) with energy given by the expression

$$E = \epsilon(\sigma_1 + \sigma_2) + J\sigma_1\sigma_2.$$

Here ϵ is the energy associated with the oxygen being bound to one of the two adsorption sites on dimoglobin, $\sigma_i = 0$ when the i^{th} site on dimoglobin is unoccupied, and $\sigma_i = 1$ when it is occupied.

(a) Evaluate the partition function for this model within the canonical ensemble.

(b) Derive expressions for the probabilities p_0 , p_1 , and p_2 corresponding to occupancy 0, 1, and 2, on the dimoglobin respectively. What is the average occupation number of oxygen molecules $\langle N \rangle$ on the dimoglobin molecule?

7. QM-1

A scanning tunneling microscope has a sharp metallic tip. The tip is scanned near the surface of a metal with constant average distance of 0.5 nm between the tip and the surface. The surface of the metal consists of atoms that have regular positions with atomic height variation of 0.05 nm.

- (1) What is the tunneling probability for an electron as a function of the separation?
- (2) Estimate the relative change (%) of the tunneling current as the tip is scanned across the metallic surface. Assume the tip and the metal are made of the same solid with a work function of 4 eV.

8. QM-2

Two point particles of mass M each and with electric charges $+Q$ and $-Q$ are joined by a massless rod of length R whose center is fixed in place but which is free to rotate in three dimensions.

- (a) (2 points) Find the energy eigenvalues of this quantum system.

If the system is initially in its ground state, and then a small uniform electric field $\vec{\mathbf{E}} = E\hat{\mathbf{z}}$ is applied, it will begin to oscillate between the ground state and the lowest excited state which can be found by first order perturbation theory.

- (b) (3 points) Write the normalized wave functions of the ground state and that excited state,
- (c) (3 points) Calculate (with the correct sign) the amount by which the ground state energy is changed (to order E^2), and
- (d) (2 points) Compute the angular frequency of the oscillation between the two states (to order E^0).

9. QM-3

Consider 2-dimensional oscillator with Hamiltonian

$$H = \frac{1}{2}(p_x^2 + p_y^2) + \frac{1}{2}(1 + \delta \cdot x \cdot y)(x^2 + y^2)$$

where $\hbar = 1$ and $\delta = \text{const} \ll 1$.

(a) (3 points) Find wavefunctions and energies for three (3) lowest states when $\delta = 0$.

(b) (7 points) Use first order degenerate or non-degenerate perturbation theory to find wavefunctions and energies for three (3) lowest states when $\delta \neq 0$.

10. QM-4

Consider a hydrogen atom in its ground state. The following time dependent uniform electric field is turned on at $t = 0$,

$$\vec{E} = \vec{E}_0 e^{-t/\tau}$$

Use first-order time-dependent perturbation theory to find the probabilities for the atom to be found in each of the 2p states, and the 2s state, at late times. Use symmetry arguments and the Wigner-Ekharth theorem to simplify your answer as much as possible.

Hint: the wavefunctions in spherical polar coordinates are

$$\begin{aligned}\psi_{100} &= \frac{1}{\sqrt{\pi}a_0^{3/2}} e^{-r/a_0} \\ \psi_{200} &= \frac{1}{4\sqrt{2\pi}a_0^{3/2}} \left(2 - \frac{r}{a_0}\right) e^{-r/2a_0} \\ \psi_{210} &= \frac{1}{4\sqrt{2\pi}a_0^{3/2}} \frac{r}{a_0} e^{-r/2a_0} \cos \theta \\ \psi_{21\pm 1} &= \frac{1}{8\sqrt{2}a_0^{3/2}} \frac{r}{a_0} e^{-r/2a_0} \sin \theta e^{\pm i\phi}\end{aligned}$$

with $a_0 = \frac{\hbar^2}{me^2}$. You may use the following integral

$$\int_0^\infty dr r^n e^{-r/a} = a^{n+1} n!$$